

# TD 20 correction

## Limites de fonctions

### Exercice 2.

$$\lim_{x \rightarrow 0^+} \left[ \frac{1}{x} \right] = +\infty$$

$$\lim_{x \rightarrow 0^+} x \left[ \frac{1}{x} \right] = 1$$

$$\lim_{x \rightarrow 0^+} x^2 \left[ \frac{1}{x} \right] = 0$$

### Exercice 3.

$$\sqrt{x + \sqrt{x}} - \sqrt{x} = \sqrt{x} \left( \sqrt{1 + \frac{1}{\sqrt{x}}} - 1 \right) \underset{x \rightarrow +\infty}{\sim} \sqrt{x} \times \frac{1}{2\sqrt{x}} \rightarrow \frac{1}{2}$$

$$\lim_{x \rightarrow +\infty} \frac{x^2 - |x|}{x^2 + |x|} = 1$$

### Exercice 4.

$$x^3 + 3x^2 - 5x + 2 \underset{x \rightarrow 0}{\sim} 2$$

$$\frac{1}{x^3} + \frac{3}{x^2} - \frac{5}{x} + 2 \underset{x \rightarrow 0}{\sim} \frac{1}{x^3}$$

$$\ln(\cos(x)) = \ln(1 + (\cos(x) - 1)) \underset{x \rightarrow 0}{\sim} -\frac{x^2}{2}$$

$$\sqrt{\cos(x^2)} - 1 = \sqrt{1 + (\cos(x^2) - 1)} - 1 \underset{x \rightarrow 0}{\sim} -\frac{x^4}{4}$$

$$e^{\tan(x)} - e^{\sin(x)} = e^{\sin(x)} \left( e^{\sin(x) \left( \frac{1}{1 + (\cos(x) - 1)} - 1 \right)} - 1 \right) \underset{x \rightarrow 0}{\sim} -\frac{x^3}{2}$$

### Exercice 5.

$$2x - \sqrt{4x^2 - x + 1} = 2x \left( 1 - \sqrt{1 - \frac{1}{4x} + \frac{1}{4x^2}} \right) \underset{x \rightarrow +\infty}{\sim} \frac{1}{4}$$

$$x \ln(1 + x^2) - 2x \ln(x) = x \ln \left( 1 + \frac{1}{x^2} \right) \underset{x \rightarrow +\infty}{\sim} \frac{1}{x}$$

$$\tan \left( \frac{1}{x} \right) - \sin \left( \frac{1}{x} \right) = \sin \left( \frac{1}{x} \right) \left( \frac{1}{1 + (\cos(1/x) - 1)} - 1 \right) \underset{x \rightarrow +\infty}{\sim} \frac{x^3}{2}$$